

EXTENDED AXION ELECTRODYNAMICS: OPTICAL ACTIVITY INDUCED BY NONSTATIONARY DARK MATTER

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Abstract

We establish a new self-consistent Einstein-Maxwell-axion model based on the Lagrangian, which is linear in the pseudoscalar (axion) field and its four-gradient and includes the four-vector of macroscopic velocity of the axion system as a whole. We consider extended equations of the axion electrodynamics, modified gravity field equations, and discuss non-stationary effects in the phenomenon of optical activity induced by axions.

1 Introduction

Axions (pseudo-Goldstone bosons) are considered to be Weakly Interacting Massive Particles (WIMPs) appearing as a result of spontaneous phase transition predicted by Peccei and Quinn [1]. These (hypothetic) particles can play a fundamental role in the formation of Dark Matter, whose contribution into the Universe energy balance is estimated to be about 23 % (see, e.g., [2] - [5]). The axion electrodynamics established by Weinberg and Wilczek [6]-[8] gives us a new instrument for the Dark Matter investigation, since the model of coupling between photons and pseudoscalar (axion) field proposed by Ni in [9] predicts the effect of polarization rotation, when the electromagnetic waves travel through the axion system (see, e.g., [10]-[13]). The optical activity is the best known but not unique phenomenon, which can be induced by axions in the electrodynamic systems. We expect that birefringence, dynamo-optical, etc. effects, which are well-known in the classical electrodynamics of moving media, can be found in axion-photons systems as well. Keeping in mind this idea, we intend to establish a number of new models unified terminologically by the common title "Extended Axion Electrodynamics". Non-minimal extension of the Einstein-Maxwell-axion theory [14] was the first step in that direction. In this short note we discuss the class of models, which satisfy the following requirements: first, the

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electrodynamics is linear; second, the pseudoscalar field equation is of the second order in derivatives; third, the cross-terms in the Lagrangian are linear in the pseudoscalar field and its four-gradient; fourth, the Lagrangian of the model includes the macroscopic velocity four-vector of the system as a whole, but does not contain its derivatives.

2 Extended model of the axion-photon coupling

2.1 On the Lagrangian of the extended model

The *standard* Einstein-Maxwell-axion model is based on the Lagrangian formalism with the action functional

$$S_{(0)} = \int d^4x \sqrt{-g} \mathcal{L}_0, \quad (1)$$

$$\mathcal{L}_{(0)} = \frac{R+2\Lambda}{\kappa} + \frac{1}{2}F_{mn}F^{mn} + \frac{1}{2}\phi F_{mn}^*F^{mn} + \Psi_0^2 \left[-\nabla_m\phi\nabla^m\phi + V(\phi^2) \right]. \quad (2)$$

Here, g is the determinant of the metric tensor g_{ik} , ∇_m is the covariant derivative, R is the Ricci scalar, $\kappa \equiv \frac{8\pi G}{c^4}$ is the Einstein coupling constant, Λ is the cosmological constant. The Maxwell tensor F_{mn} is given by

$$F_{mn} \equiv \nabla_m A_n - \nabla_n A_m, \quad \nabla_k F^{*ik} = 0, \quad (3)$$

where A_m is an electromagnetic potential four-vector; $F^{*mn} \equiv \frac{1}{2}\epsilon^{mnpq}F_{pq}$ is the tensor dual to F_{pq} ; $\epsilon^{mnpq} \equiv \frac{1}{\sqrt{-g}}E^{mnpq}$ is the Levi-Civita tensor, E^{mnpq} is the absolutely antisymmetric Levi-Civita symbol with $E^{0123} = 1$. It is the third term in the Lagrangian that describes the pseudoscalar-photon interaction [9]. The symbol ϕ stands for a pseudoscalar field, this quantity being dimensionless. The axion field itself, Φ , is considered to be proportional to this quantity $\Phi = \Psi_0\phi$ with a phenomenological constant Ψ_0 . The function $V(\phi^2)$ describes the potential of the pseudoscalar field.

Now we extend the Lagrangian (2) by the terms, which are quadratic in the Maxwell tensor F_{mn} , are linear in ϕ or in $\nabla_k\phi$, and contain the normalized four-vector U^k ($U^k U_k = 1$). The quantity U^k describes the macroscopic velocity of the axion system as a whole, and may be chosen as the time-like eigen-vector of the stress-energy tensor of the pseudoscalar (axion) field. In order to list all the

irreducible scalars, which satisfy these requirements, let us remind the important identity

$$F^{ik}F_{kj}^* = \frac{1}{4}\delta_j^i F^{mn}F_{mn}^* . \quad (4)$$

Clearly, all the invariants, which we could construct using g_{ij} , F^{ik} , F_{mn}^* , U^k , as well as, ϕ or $\nabla_k\phi$, definitely contain at least one convolution of the type $F^{ik}F_{kj}^*$. Thus, it is easy to check that due to (4) all the new terms in the extended Lagrangian can be reduced to the invariant

$$\mathcal{L}_{(\text{int})} = \frac{1}{2}\nu F^{mn}F_{mn}^* U^k \nabla_k \phi , \quad (5)$$

where ν is some new coupling constant introduced phenomenologically.

2.2 Extension of the axion electrodynamics

The variation of the action functional containing the sum of Lagrangians $\mathcal{L}_{(0)} + \mathcal{L}_{(\text{int})}$ with respect to the four-vector potential A_i gives the equations of axion electrodynamics

$$\nabla_k H^{ik} = 0 . \quad (6)$$

Here the excitation tensor H^{ik} is given by the term

$$H^{ik} = F^{ik} + F^{*ik} (\phi + \nu \mathcal{D}\phi) , \quad (7)$$

and $\mathcal{D} = U^k \nabla_k$ is the convective derivative. Using the linear constitutive equations

$$H^{ik} = C^{ikmn} F_{mn} , \quad (8)$$

we readily obtain that the linear response tensor C^{ikmn} now takes the form

$$C^{ikmn} = \frac{1}{2} (g^{im}g^{kn} - g^{in}g^{km}) + \frac{1}{2}\epsilon^{ikmn} (\phi + \nu \mathcal{D}\phi) . \quad (9)$$

This means that the dielectric permittivity and magnetic impermeability tensors of the axion-photon system are the same as in vacuum, i.e.,

$$\varepsilon^{im} = 2C^{ikmn}U_k U_n = \Delta^{im} , \quad (10)$$

where $\Delta^{im} = g^{im} - U^i U^m$ is the projector, and

$$(\mu^{-1})_{pq} = -\frac{1}{2}\eta_{pik}C^{ikmn}\eta_{mnq} = \Delta_{pq} , \quad (11)$$

where $\eta_{pik} \equiv \epsilon_{pikj}U^j$. The tensor of magneto-electric coefficients

$$\nu_p^m = \eta_{pik}C^{ikmn}U_n = -\Delta_p^m (\phi + \nu \mathcal{D}\phi) , \quad (12)$$

describing optical activity effects (see, e.g., [15]) is now characterized by additional term linear in $\mathcal{D}\phi$.

2.3 Pseudoscalar field evolution

Variation of the extended action functional with respect to the pseudoscalar field ϕ gives the equation

$$\nabla_k \nabla^k \phi + \phi V'(\phi^2) = \frac{1}{4\Psi_0^2} [F_{mn}^* F^{mn} (\nu\theta - 1) + \nu \mathcal{D}(F_{mn}^* F^{mn})] , \quad (13)$$

where $\theta \equiv \nabla_k U^k$ is the expansion scalar of the velocity field, and the prime denotes the derivative of the potential $V(\phi^2)$ with respect to the argument.

2.4 Gravity field equations

Modified Einstein's equations obtained by the variation of the extended action functional with respect to the metric g^{pq} can be presented in the form

$$R_{pq} - \frac{1}{2} g_{pq} R = \Lambda g_{pq} + \kappa [T_{pq}^{(EM)} + T_{pq}^{(A)} + \nu T_{pq}^{(*)}] . \quad (14)$$

Here the stress-energy tensor of the electromagnetic field

$$T_{pq}^{(EM)} = \frac{1}{4} g_{pq} F_{mn} F^{mn} - F_{pm} F_q^m \quad (15)$$

and the stress-energy tensor of the pure axionic field

$$T_{pq}^{(A)} = \Psi_0^2 \left\{ \nabla_p \phi \nabla_q \phi - \frac{1}{2} g_{pq} [\nabla_m \phi \nabla^m \phi - V(\phi^2)] \right\} \quad (16)$$

are presented by the well-known terms. The tensor

$$T_{pq}^{(*)} = -\frac{1}{8} F^{mn} F_{mn}^* (U_p \nabla_q \phi + U_q \nabla_p \phi) , \quad (17)$$

describes a principally new source-term in the right-hand side of the gravity field equations. Let us mention that the term $\nu T_{pq}^{(*)}$ is obtained by the variation of the term with the interaction Lagrangian (5) by using the formula

$$\delta U^i = \frac{1}{4} \delta g^{pq} (U_p \delta_q^i + U_q \delta_p^i) \quad (18)$$

for the variation of the velocity four-vector (see [16] for details). The standard interaction term $\frac{1}{2} \sqrt{-g} \phi F^{mn} F_{mn}^* = \frac{1}{4} \phi E^{ikmn} F_{ik} F_{mn}$ does not contribute to the stress-energy tensor in the process of variation with respect to the metric. Thus, the appearance of the term (17) is a new event in the modeling of the gravity field of the photon-axion system.

2.5 An example of application

Let us consider the propagation of *test* electromagnetic wave coupled to the axionic subsystem of the Dark Matter in the spatially homogeneous FLRW-type spacetime with the scale factor $a(t)$. Let the electromagnetic wave propagate in the direction $0x$ and be characterized by the potential four-vector $A_i = \delta_i^2 A_2(t, x) + \delta_i^3 A_3(t, x)$. The equations of axion electrodynamics can be now reduced to

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{a^2} \frac{\partial^2}{\partial x^2} + H \frac{\partial}{\partial t} \right] A_2 = -\frac{2\dot{\Theta}}{a} \frac{\partial}{\partial x} A_3, \quad (19)$$

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{a^2} \frac{\partial^2}{\partial x^2} + H \frac{\partial}{\partial t} \right] A_3 = \frac{2\dot{\Theta}}{a} \frac{\partial}{\partial x} A_2, \quad (20)$$

where $H(t) = \frac{\dot{a}}{a}$ is the Hubble function and

$$\Theta(t) \equiv \frac{1}{2} [\phi(t) + \nu \dot{\phi}(t)], \quad (21)$$

(in the cosmological context we use the units with $c = 1$). Clearly, when $\nu = 0$ and $\dot{\phi} \neq 0$ the electromagnetic wave can not keep linear polarization, however, in case when $\nu \neq 0$ and the pseudoscalar field evolves exponentially $\phi(t) \propto \exp\left[-\frac{t}{\nu}\right]$, it can be possible. In the approximation of short wavelengths $k \gg H$ the solution of (19), (20) for the circularly polarized wave has the form

$$A_2 = -A_0 \sin[W - \varphi(t)], \quad A_3 = A_0 \cos[W - \varphi(t)], \quad (22)$$

$$W = W(t_0) + k \left[\int_{t_0}^t \frac{dt'}{a(t')} - x \right], \quad \varphi(t) \equiv \Theta(t) - \Theta(t_0), \quad (23)$$

where k is a constant reciprocal to the wavelength. On the one hand, the quantity $\varphi(t)$ is expressed in terms of ϕ and $\dot{\phi}$ (see (23) and (21)) and describes the rotation angle of the polarization vector of the electromagnetic wave traveling through the axion system; it can be studied in optical experiments. On the other hand, it is well-known that in the cosmological context the function $\dot{\phi}$ can be represented in terms of the Dark Matter energy-density \mathcal{E} and pressure \mathcal{P} as follows

$$\dot{\phi} = \pm \frac{1}{\Psi_0} \sqrt{\mathcal{E}(t) + \mathcal{P}(t)}. \quad (24)$$

Thus, the extended axion electrodynamics can be considered as a tool for investigation of the nonstationary effects in the evolution of the axionic Dark Matter.

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